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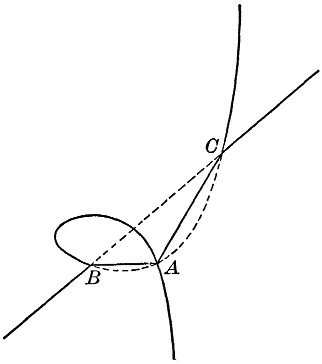
The tangents to (C_3) at A are the bisectors C, C'. The foot of the perpendicular from A upon BC belongs to (C_3).

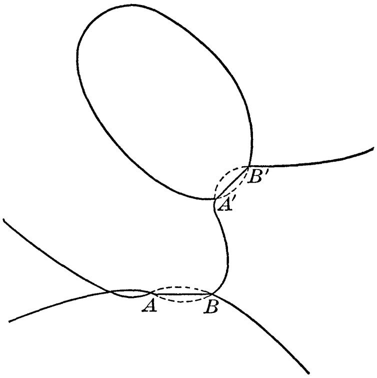
Remarks, I. In order that all the points of (C_3) shall belong to the required locus the condition of " equal angles " must be replaced by " equal positive angles." Otherwise only those parts of (C_3) that are included in the angle BAC and in its vertical angle, will posses the required property.

II. If $AC = AB$, the cubic (C_3) degenerates into the bisector of the angle BAC and the circumference of the circle circumscribed about the triangle ABC.

III. The line of centers OO' envelops a parabola similar to the parabola (P), the point A being the center of similitude.

Editorial Note.—The formulas of analytic geometry and the procedure of synthetic geometry are both such that continuous deformations do not destroy their validity. The word " subtend," having physical connotations, is not so flexible.





The locus in general, for the usual meaning of subtend, is made up of parts of two cubic curves forming a continuous curve in the projective plane without a continuous derivative. In the special case one cubic degenerates and the real part is the line BC. The heavy curves in the accompanying cuts sufficiently show the locus.

443. Proposed by C. N. SCHMALL, New York City.

A quadrilateral of any shape whatever is divided by a transversal into two quadrilaterals. The diagonals of the original figure and those of the two resulting (smaller) figures are then drawn. Show that their three points of intersection are collinear.

SOLUTION BY LAENAS G. WELD, Pullman, Ills.

[The word "(smaller)" should be omitted, as it destroys the generality of the proposition.]

Using trilinear coördinates, let ABC be the triangle of reference and designate the points in which the lines $l\alpha + m\beta + n\gamma = 0$ and $p\alpha + q\beta + r\gamma = 0$ cut the sides a, b, c by L, M, N and P, Q, R, respectively. Then $BCMN$, $BCQR$ and $QMNR$ are the three quadrilaterals in question. The equations of the diagonals are readily written as follows:

$$BM\colon\ l\alpha + n\gamma = 0, \qquad BQ\colon\ p\alpha + r\gamma = 0,$$

$$CN\colon\ l\alpha + m\beta = 0, \qquad CR\colon\ p\alpha + q\beta = 0,$$

$$RM\colon\ lp\alpha + lq\beta + np\gamma = 0,$$

$$QN\colon\ lp\alpha + mp\beta + lr\gamma = 0.$$

At the intersections of BM and CN, of BQ and CR, and of RM and QN, the coördinate ratios are respectively:

$$\begin{array}{lccccc} \alpha_1 : \beta_1 : \gamma_1 :: & -mn & : & nl & : & lm, \\ \alpha_2 : \beta_2 : \gamma_2 :: & -qr & : & rp & : & pq, \\ \alpha_3 : \beta_3 : \gamma_3 :: & l^2rq - p^2mn & : & p^2nl - l^2rp & : & p^2lm - l^2pq. \end{array}$$

The vanishing of the determinant of these three sets of coördinates proves the proposition.

Also solved by NATHAN ALTSHILLER, H. C. FEEMSTER, and F. M. MORGAN.

444. Proposed by S. A. COREY, Hiteman, Iowa.

Let $ABCDE$ be a pentagon, plane or gauche, with sides AB, BC, CD, DE, and EA. Bisect BC and DE in H and K respectively. Extend AB from B to B', and AE from E to E'. On AB' take sects AP and AV, and on AE' take sects AL and AT. Draw AD, AC, AH, AK, and DT. Let a, b, c, and d equal AL/AE, AT/AE, AV/AB, and AP/AB, respectively. Extend (or contract) AC from C to W, and AD from D to S, making $AW = a \times AC$ and $AS = d \times AD$. Draw LM and PN parallel to, and of the same currency as, AD and AC respectively, and of lengths $c \times AD$ and $b \times AC$, respectively. Draw AM, AN, ST, and WV. Draw DQ and VX parallel to, and of the same currency as, CB and TS, respectively. We are to prove that $2(ad + bc)(AK \times AH \times \cos KAH + KE \times HC \times \cos QDK) = AM \times AN \times \cos MAN + TS \times VW \times \cos WVX$.

SOLUTION BY THE PROPOSER.

Let KE, HB, AH and AK, in the proposed figure, be represented by the vectors x, y, z, and w, respectively. Then by vector addition, $w + x = AE$, $w - x = AD$, $z + y = AB$, $z - y = AC$; by construction, $\angle ST \cdot WV = \angle WVX$, $\angle KE \cdot HB = \angle QDK$, $(w + x)a = AL$, $(w - x)c = LM$, $(z + y)d = AP$, $(z - y)b = PN$, $(w + x)b = AT$, $(w - x)d = AS$, $(z + y)z = AV$, and $(z - y)a = AW$; also by vector addition, $(w + x)a + (w - x)c = AM$, $(z + y)d + (z - y)b = AN$, $(w + x)b - (w - x)d = ST$, and $(z + y)c - (z - y)a = WV$.

Consider now the algebraic identity,

$$\begin{aligned} &(w + x)a + (w - x)c][(z + y)d + (z - y)b] \\ &\quad + [(w + x)b - (w - x)d][(z + y)c - (z - y)a] = 2(ad + bc)(wz + xy) \end{aligned} \tag{1}$$

and note that when fully expanded each term is of the second degree in x, y, z, and w. Also note that the identity may be written

$$AM \times AN + ST \times WV = 2(ad + bc)(AK \times AH + KE \times HB) \tag{2}$$

if we substitute vectors as above indicated.